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Atomic squeezing in assembly of two two-level atoms interacting with a single mode coherent radiation

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Abstract. Saito and Ueda [Phys. Rev. A **59**, 3959 (1999)] studied atomic and radiation squeezing in interaction of a single mode coherent state $|\alpha\rangle$ of radiation with two excited two-level atoms, using the Jaynes Cummings Hamiltonian. They considered α real and studied squeezing of the Dicke operator S_x using the Kitagawa-Ueda criterion for squeezing and coupling times less than or nearly equal to $|\alpha|^{-1}$. We obtain results to all orders in coupling time for atoms, which are initially in (i) fully excited, (ii) superradiant or in (iii) ground states and obtain more general results. We use our recently reported criterion for atomic squeezing, of which the Kitagawa-Ueda criterion is a special case, and obtain a much stronger (nearly 95%) atomic squeezing than that (nearly 1.1%) reported by Saito and Ueda.

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QICS. 15.10.-p Quantum optics: Physical qubits

1 Introduction

Squeezed radiation states in quantum optics are distinguished by the property that the quantum fluctuations in one of the field quadratures are less than those associated with coherent light or vacuum [1,2]. Earlier [3,4], squeezing was largely of academic interest because of this quantum feature. But now, it is realized that squeezing can not only help to increase signal to noise ratio in one quadrature component [1,5] but also in optical communication [6], gravitational wave detection [7] and in the field of quantum information [8]. Quantum mechanical correlation between photons established through non-linear interaction play an essential role in the generation of squeezed states of light. The generation of squeezed states has been predicted in a number of non-linear optical systems. Among them are multi photon amplifier [4], parametric amplifier [3,9], resonance fluorescence [10], four-wave mixing [2,5] and interaction of two and three level systems [11,12] with a coherent field in Jaynes and Cummings model.

In analogy to squeezing of light [1,2], squeezing of spin components has been defined by several authors [10,13–19]. The earliest definition of atomic squeezing is due to Walls and Zoller [10], who wrote the condition for squeezing in S_x and S_y component in the forms $\langle(\Delta S_x)^2\rangle < |\langle S_z\rangle|/2$ and $\langle(\Delta S_y)^2\rangle < |\langle S_z\rangle|/2$ respectively. These definitions have been used by several authors [13,14]. Later several authors gave alternate definitions of atomic squeezing, in the form $\xi < 1$, where ξ, the squeezing parameter or factor, has been defined differently by different set of authors. Wineland et al. [15], who studied resolution in spectroscopic experiments on N two-level atoms, defined atomic squeezing for spin components in a plane normal to mean spin and wrote, $\xi = (2S)^{1/2}\langle(\Delta S_\perp)\rangle/|\langle\bar{S}\rangle|$, where $\Delta S_\perp$ denotes the smallest uncertainty of a spin component perpendicular to mean spin vector $\bar{S}$. This gives a measure of the quantum noise in a direction perpendicular to the mean value of the total spin. Kitagawa and Ueda [16] also defined squeezing of spin components normal to mean spin and wrote the squeezing parameter as, $\xi = \langle(\Delta S_\perp)\rangle/|\langle\bar{S}\rangle/2|^{1/2}$. This definition was also used by Saito and Ueda [17]. Sorensen et al. [18] proposed the parameter for defining the atomic squeezing, $\xi = \sqrt{N\langle(\Delta S_x)^2\rangle/\langle S_y\rangle^2 + \langle S_z\rangle^2}$.

Walls and Zoller [10], Saito and Ueda [17], Wang and Sanders [19] have shown that squeezing of quadrature amplitude $q_\theta = (ae^{-i\theta} + a^+e^{i\theta})/\sqrt{2}$ depends on squeezing of spin component $S_{\theta+\pi/2} \equiv -S_x \sin\theta + S_y \cos\theta$. Spin components in the x–y-plane control radiation squeezing and assume more significance than other spin component. As

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a consequence, one should study the squeezing of all spin components in the x–y-plane and not only that which is perpendicular to the mean spin $\langle \boldsymbol{S} \rangle$, as has been done by Wineland et al. [15], Kitagawa and Ueda [16] and Saito and Ueda [17]. A generalization of the definition of atomic squeezing so as to define squeezing of all spin components is necessary and has been done recently by the authors [20].

The present authors defined recently the squeezing parameter [20] for components in the x–y-plane as, $\mathsf{S}_\theta = 2\langle(\Delta S_\theta)^2\rangle/[\langle S_{\theta+\pi/2}\rangle^2 + \langle S_z\rangle^2]^{1/2}$ where, $S_\theta = S_x\cos\theta + S_y\sin\theta$. *This is a natural generalization of the earliest and simplest definition of Walls and Zoller [10] and the Kitagawa-Ueda definition is a special case of this definition when direction of mean spin is perpendicular to* θ-direction. Saito and Ueda [17] used the definition of Kitagawa and Ueda [16] to study extensively atomic squeezing in interaction of a system of two excited two-level atoms with a single coherent mode of radiation. We also study the same interaction but with (i) atoms fully excited, (ii) superradiant or in (ii) ground state and find more extensive results, of which some of the Saito-Ueda results are subsets. Also, we report much stronger (∼95%) atomic squeezing as compared to the Saito-Ueda result (∼1.1%). The experimental detection has been discussed by Saito-Ueda [17] and Polzik [21] in detail and we do not want to add anything to it.

2 The exact time evolution operator

Consider a system of two two-level atoms interacting with a single resonant mode of radiation with zero detuning. If the atoms are located in a region small in comparison with the wavelength of the field, but not so small so as to make them interact directly with each other, the Hamiltonian of the system is given in the natural system of units ($\hbar = 1$) and the dipole and rotating wave approximations by [22]

$$H = H_0 + H_I, \quad H_0 = H_F + H_A, \quad H_F = \omega a^+ a,$$
$$H_A = \omega S_z, \quad H_I = g(aS_+ + a^+ S_-). \tag{1}$$

Here, subscripts F, A, and I refer to field, atoms and interaction, g is coupling constant and $S_{\pm,z}$ are the Dicke's collective atom operators [23]. Similar Hamiltonian for a single two-level atom and single mode radiation was solved exactly independently by Stenholm [24] and Prakash, Chandra and Vachaspati [25].

If $|u\rangle_i$ and $|l\rangle_i$ are the interacting upper and lower energy states of the ith ($i = 1$, 2) two-level atoms,

$$S_\pm = \sum_{i=1,2} S_{\pm i}; \tag{2a}$$

$$S_z = \sum_{i=1,2} S_{zi}; \tag{2b}$$

$$S_\pm \equiv S_x \pm iS_y, \tag{2c}$$

$$S_{+i} = |u\rangle_i\ {}_i\langle l|; \tag{3a}$$
$$S_{-i} = |l\rangle_i\ {}_i\langle u|; \tag{3b}$$
$$S_z = \tfrac{1}{2}[|u\rangle_i\ {}_i\langle u| - |l\rangle_i\ {}_i\langle l|]. \tag{3c}$$

$S_\pm$ and S_z satisfy the commutation relations,

$$[S_+, S_-] = 2S_z, \quad [S_z, S_\pm] = \pm S_\pm. \tag{4}$$

In the truncated Hilbert space, the atomic system is described by the eigenstate $|j, m\rangle$ defined by,

$$S^2\,|j, m\rangle = j(j+1)\,|j, m\rangle\,; \quad S_z\,|j, m\rangle = m\,|j, m\rangle\,;$$
$$S^2 = \frac{1}{2}\,[S_+S_- + S_-S_+] + S_z^2. \tag{5}$$

For a system of two two-level atoms, $j = 1$, with $m = 1$, 0, −1 and $j = 0$, with $m = 0$. These states have the property, $S_\pm|j, m\rangle = \sqrt{(j \mp m)(j \pm m + 1)}|j, m \pm 1\rangle$. Since $[S^2, H] = 0$, quantum number j does not change in interaction. The $j = 0$ state, $|0, 0\rangle$, does not interact as $S_\pm|0, 0\rangle = 0$ giving $H_I|0, 0\rangle = 0$. If $j = 1$ initially, we have to consider states $|1, 1\rangle$, $|1, 0\rangle$ and $|1, -1\rangle$ only. In the interaction picture, the interaction Hamiltonian can be written as

$$H_I = \sqrt{2}\,gF, \qquad F = \begin{pmatrix} 0 & a & 0 \\ a^+ & 0 & a \\ 0 & a^+ & 0 \end{pmatrix}. \tag{6}$$

As explained in reference [20], this leads exactly to the time evolution operator,

$$U_I = \exp(-iH_I t)$$
$$= \begin{pmatrix} 1{+}(N{+}1)C(N{+}1) & -iS(N{+}1)a & C(N{+}1)a^2 \\ -ia^+S(N{+}1) & \cos(gt\sqrt{4N{+}2}) & -iS(N)a \\ a^{+2}C(N{+}1) & -ia^+S(N) & 1{+}NC(N{-}1) \end{pmatrix}, \tag{7}$$

where, $N = a^+a$ is the number operator and

$$C(N) \equiv \{\cos(gt\sqrt{4N+2}) - 1\}/(2N+1);$$
$$S(N) \equiv \{\sin(gt\sqrt{4N+2})\}/\sqrt{2N+1}. \tag{8}$$

3 Atomic squeezing in interaction of two two-level atoms with a single mode coherent radiation

Instead of considering squeezing of spin components S_x and S_y separately, let us consider a more general operator,

$$S_\theta = S_x\cos\theta + S_y\sin\theta. \tag{9}$$

Commutation relation $[S_\theta, S_{\theta+\pi/2}] = iS_z$ gives

$$\langle(\Delta S_\theta)^2\rangle\langle(\Delta S_{\theta+\pi/2})^2\rangle \geqslant \frac{1}{4}|\langle S_z\rangle|^2. \tag{10}$$

Commutation relations $[S_\theta, S_{\theta+\pi/2}] = iS_z$ and $[S_\theta, S_z] = -iS_{\theta+\pi/2}$, indicate squeezing for S_θ, if $\langle(\Delta S_\theta)^2\rangle < |\langle S_z\rangle/2|$ or $\langle(\Delta S_\theta)^2\rangle < |\langle S_{\theta+\pi/2}\rangle/2|$. We can obtain *the most general criterion for squeezing* of the operator S_θ by considering, in place of the triad of operators, ($S_\theta, S_{\theta+\pi/2}$ and S_z), the triad, (S_θ, $S_{\theta+\pi/2,\phi}$ and $S_{\theta+\pi/2,\phi+\pi/2}$) with

$$S_{\theta+\pi/2,\phi} \equiv S_{\theta+\pi/2}\cos\phi + S_z \sin\phi,$$
$$S_{\theta+\pi/2,\phi+\pi/2} \equiv -S_{\theta+\pi/2}\sin\phi + S_z\cos\phi, \qquad (11)$$

and an arbitrary ϕ. These operators give $[S_\theta, S_{\theta+\pi/2,\phi}] = iS_{\theta+\pi/2,\phi+\pi/2}$, $[S_\theta, S_{\theta+\pi/2,\phi+\pi/2}] = -iS_{\theta+\pi/2,\phi}$ and therefore the uncertainty relations,

$$\langle(\Delta S_\theta)^2\rangle\langle(\Delta S_{\theta+\pi/2,\phi})^2\rangle \geqslant \frac{1}{4}|\langle S_{\theta+\pi/2,\phi+\pi/2}\rangle|^2, \qquad (12a)$$

$$\langle(\Delta S_\theta)^2\rangle\langle(\Delta S_{\theta+\pi/2,\phi+\pi/2})^2\rangle \geqslant \frac{1}{4}|\langle S_{\theta+\pi/2,\phi}\rangle|^2. \qquad (12b)$$

One may call S_θ squeezed if

$$\langle(\Delta S_\theta)^2\rangle < \frac{1}{2}|\langle S_{\theta+\pi/2,\phi+\pi/2}\rangle|$$
$$\text{and/or } \langle(\Delta S_\theta)^2\rangle < \frac{1}{2}|\langle S_{\theta+\pi/2,\phi}\rangle|. \qquad (13)$$

Equation (11) shows that the values of $|\langle S_{\theta+\pi/2,\phi}\rangle|$ and $|\langle S_{\theta+\pi/2,\phi+\pi/2}\rangle|$, as ϕ is varied, lies between 0 and $[\langle S_z\rangle^2 + \langle S_{\theta+\pi/2}\rangle^2]^{1/2}$. *The most general criterion for squeezing is, therefore,*

$$\left\langle(\Delta S_\theta)^2\right\rangle < \frac{1}{2}\left[\left\langle S_z\right\rangle^2 + \left\langle S_{\theta+\pi/2}\right\rangle^2\right]^{1/2}, \qquad (14)$$

because, *if this relation holds, then one can always find separate intervals for* ϕ for holding of the two equations (13). In case these two intervals for ϕ overlap, in the region of overlap, both of equations (13) are satisfied and both components, S_θ and $S_{\theta+\pi/2}$ are squeezed [20].

It should be noted that Saito and Ueda [17] define the component of $\boldsymbol{S}$ [26] normal to mean spin vector $\langle\boldsymbol{S}\rangle$ as $S(\hat{\mathbf{n}},\chi) \equiv \exp(-i\chi\boldsymbol{S}\cdot\hat{\mathbf{n}})(\boldsymbol{S}\cdot\hat{\mathbf{m}})\exp(i\chi\boldsymbol{S}\cdot\hat{\mathbf{n}})$, where $\hat{\mathbf{n}} = \langle\boldsymbol{S}\rangle/|\langle\boldsymbol{S}\rangle|$, $\hat{\mathbf{m}} = \hat{\mathbf{n}}\times\hat{\mathbf{e}}_{\mathbf{z}}/|\hat{\mathbf{n}}\times\hat{\mathbf{e}}_{\mathbf{z}}|$ and χ is the angle measured in the plane normal to $\hat{\mathbf{n}}$ measured from $\hat{\mathbf{m}}$. They consider commutation relation $[\mathrm{S}(\hat{\mathbf{n}},\chi), \mathrm{S}(\hat{\boldsymbol{n}},\chi+\frac{1}{2}\pi)] = i(\boldsymbol{S}\cdot\hat{\mathbf{n}})$, and write the uncertainty relation as [27]

$$\langle[\Delta S(\hat{\mathbf{n}},\chi)]^2\rangle\langle[\Delta S(\hat{\mathbf{n}},\chi+\pi/2)]^2\rangle \geqslant \frac{1}{4}|\langle\boldsymbol{S}\rangle|^2, \qquad (15)$$

since $|\langle\boldsymbol{S}\rangle| = |\langle\boldsymbol{S}\cdot\hat{\mathbf{n}}\rangle|$ in this case, and use the criterion for squeezing, $\mathsf{S}_{\mathrm{SU}} < 1$, with

$$\mathsf{S}_{\mathrm{SU}} = \langle[\Delta\mathrm{S}(\hat{\mathbf{n}},\chi)]^2\rangle/\frac{1}{2}|\langle\boldsymbol{S}\rangle|. \qquad (16)$$

It may be noted that if $\alpha = |\alpha|e^{i\theta_\alpha}$, using $||\alpha|e^{i\theta_\alpha}\rangle = e^{i\theta_\alpha N}||\alpha|\rangle$, we have

$$\begin{aligned}\langle f(S_\theta)\rangle &\equiv \langle\alpha|\langle 1,1|e^{iH_It}f(S_\theta)e^{-iH_It}|1,1\rangle|\alpha\rangle\\ &= \langle\alpha|\langle 1,1|e^{iH_It}e^{-i\theta(S_z+N)}f(S_x)e^{i\theta(S_z+N)}\\ &\quad\times e^{-iH_It}|1,1\rangle|\alpha\rangle\\ &= \langle|\alpha|e^{i(\theta+\theta_\alpha)}|\langle 1,1|e^{iH_It}f(S_x)\\ &\quad\times e^{-iH_It}|1,1\rangle||\alpha|e^{i(\theta+\theta_\alpha)}\rangle, \qquad (17)\end{aligned}$$

a function of $|\theta+\theta_\alpha|$. Hence results for the operator S_θ with a given value of θ say, $\theta = 0$ (i.e. $S_\theta = S_x$) but with an arbitrary value of θ_α can give the corresponding results for all values of θ and of θ_α.

For $q_\theta = (ae^{-i\theta} + a^+e^{i\theta})/\sqrt{2}$, $p_\theta = (ae^{-i\theta} - a^+e^{i\theta})/(\sqrt{2}i)$ the interaction Hamiltonian can be written as $H_I = \sqrt{2}g(q_\theta S_\theta + p_\theta S_{\theta+\pi/2})$. Squeezing of quadrature component q_θ depends on atomic squeezing of $S_{\theta+\pi/2}$ [10,17]. Hence it is important to know the value of θ for which S_θ is maximum squeezed for knowing the maximum squeezed quadrature component of radiation. To discuss squeezing, therefore, one may fixed θ_α and study variation of results with θ (as has been done here or by Saito and Ueda who took $\theta_\alpha = 0$) or fix θ and study variations with θ_α.

3.1 Case (i): both atoms are excited, initially

If both atoms are excited and radiation is in the coherent state $|\alpha\rangle$ initially, the initial state is $|\alpha\rangle|1,1\rangle$, and the final state is then obtained using equation (7) in the form,

$$\begin{aligned}|\psi\rangle &= [1+(N+1)C(N+1)]|\alpha\rangle|1,1\rangle\\ &\quad - ia^+S(N+1)|\alpha\rangle|1,0\rangle + a^{+2}C(N+1)|\alpha\rangle|1,-1\rangle. \qquad (18)\end{aligned}$$

Direct results using equations (17) and (18) are,

$$\langle S_\theta\rangle = \sqrt{2}|\alpha|\sin(\theta+\theta_\alpha)(P_1-P_2), \qquad (19)$$

$$\langle S_{\theta+\pi/2}\rangle = \sqrt{2}|\alpha|\cos(\theta+\theta_\alpha)(P_1-P_2), \qquad (20)$$

$$\langle S_z\rangle = Q_1 - Q_2, \qquad (21)$$

$$\langle S_\theta^2\rangle = \frac{1}{2} + \frac{1}{2}R_1 + |\alpha|^2\cos 2(\theta+\theta_\alpha)R_2, \qquad (22)$$

where,

$$\begin{aligned}P_1 &= \langle\alpha|(N+2)S(N+2)C(N+1)|\alpha\rangle,\\ P_2 &= \langle\alpha|[1+(N+2)C(N+2)]S(N+1)|\alpha\rangle,\\ Q_1 &= \langle\alpha|[1+(N+1)C(N+1)]^2|\alpha\rangle,\\ Q_2 &= \langle\alpha|C(N+1)a^2a^{+2}C(N+1)|\alpha\rangle,\\ R_1 &= \langle\alpha|[S(N+1)]^2(N+1)|\alpha\rangle,\\ R_2 &= \langle\alpha|[1+(N+3)C(N+3)]C(N+1)|\alpha\rangle\end{aligned}$$

and $C(N)$ and $S(N)$ are as defined by equations (8).

3.2 Case (ii): atoms are in superradiant state, initially

If atomic assembly is in the super-radiant state $|1, 0\rangle$ and radiation is in the coherent state $|\alpha\rangle$ initially, the initial state is $|\alpha\rangle|1, 0\rangle$, and the final state is then obtained using equation (7) in the form,

$$|\psi\rangle = -iS(N+1)a|\alpha\rangle|1,1\rangle + \cos(gt\sqrt{4N+2})|\alpha\rangle|1,0\rangle - ia^{+}S(N)|\alpha\rangle|1,-1\rangle. \quad (23)$$

Our direct results using equations (17) and (23) are,

$$\langle S_\theta\rangle = \sqrt{2}|\alpha|\sin(\theta+\theta_\alpha)(P_1-P_2), \quad (24)$$

$$\langle S_{\theta+\pi/2}\rangle = \sqrt{2}|\alpha|\cos(\theta+\theta_\alpha)(P_1-P_2), \quad (25)$$

$$\langle S_z\rangle = Q_1 - Q_2, \quad (26)$$

$$\langle S_\theta^2\rangle = \frac{1}{2} + \frac{1}{2}R_1 + |\alpha|^2\cos 2(\theta+\theta_\alpha)R_2 \quad (27)$$

but with,

$$P_1 = \langle\alpha|[S(N+1)\cos(gt\sqrt{4N+2})|\alpha\rangle,$$
$$P_2 = \langle\alpha|\cos(gt\sqrt{4N+6})S(N)|\alpha\rangle,$$
$$Q_1 = |\alpha|^2\langle\alpha|[S(N+1)]^2|\alpha\rangle,$$
$$Q_2 = \langle\alpha|S(N)(N+1)S(N)|\alpha\rangle,$$
$$R_1 = \langle\alpha|[\cos(gt\sqrt{4N+2})]^2|\alpha\rangle,$$
$$R_2 = \langle\alpha|S(N+2)S(N)|\alpha\rangle.$$

3.3 Case (iii): atoms are in ground state, initially

If both atoms are in ground state and radiation is in the coherent state $|\alpha\rangle$ initially, the initial state is $|\alpha\rangle|1, -1\rangle$, and the final state is then obtained using equation (7) in the form,

$$|\psi\rangle = C(N+1)a^2|\alpha\rangle|1,1\rangle - iS(N)a|\alpha\rangle|1,0\rangle + [1+NC(N-1)]|\alpha\rangle|1,-1\rangle. \quad (28)$$

Direct results using equations (17) and (28) are,

$$\langle S_\theta\rangle = \sqrt{2}|\alpha|\sin(\theta+\theta_\alpha)(P_1-P_2), \quad (29)$$

$$\langle S_{\theta+\pi/2}\rangle = \sqrt{2}|\alpha|\cos(\theta+\theta_\alpha)(P_1-P_2), \quad (30)$$

$$\langle S_z\rangle = Q_1 - Q_2, \quad (31)$$

$$\langle S_\theta^2\rangle = \frac{1}{2} + \frac{1}{2}|\alpha|^2R_1 + |\alpha|^2\cos 2(\theta+\theta_\alpha)R_2, \quad (32)$$

but with,

$$P_1 = \langle\alpha|[1+NC(N-1)]S(N)|\alpha\rangle,$$
$$P_2 = |\alpha|^2\langle\alpha|C(N+1)S(N)|\alpha\rangle,$$
$$Q_1 = |\alpha|^4\langle\alpha|[C(N+1)]^2|\alpha\rangle,$$
$$Q_2 = \langle\alpha|[1+NC(N-1)]^2|\alpha\rangle,$$
$$R_1 = \langle\alpha|[S(N)]^2|\alpha\rangle,$$
$$R_2 = \langle\alpha|[1+NC(N-1)]C(N+1)|\alpha\rangle.$$

4 Discussion of atomic squeezing

Using equation (14), we define the *squeezing factor* S_θ, by writing

$$\mathsf{S}_\theta \equiv \left\langle(\Delta S_\theta)^2\right\rangle / \frac{1}{2}\left[\langle S_z\rangle^2 + \left\langle S_{\theta+\pi/2}\right\rangle^2\right]^{1/2}, \quad (33)$$

which is a function of $|\theta + \theta_\alpha|$, $|\alpha|$ and gt. This gives atomic squeezing for a general spin component S_θ, whenever $\mathsf{S}_\theta < 1$. The Kitagawa Ueda definition equation (16) used by Saito and Ueda [17] is a special case of this definition, when their unit vector $\hat{\mathbf{n}}$ along the direction of mean spin is perpendicular to the θ-direction. For this case, $\langle S_\theta\rangle = 0$, $|\langle \boldsymbol{S}\rangle| = \sqrt{\langle S_{\theta+\pi/2}\rangle^2 + \langle S_z\rangle^2}$, and the definition equation (16) for atomic squeezing becomes identical with our definition equation (33).

Saito and Ueda considered α real (i.e. $\theta_\alpha = 0$). Since, $\langle S_y\rangle = 0$ for this case (see Eq. (20) which gives $\langle S_{\theta+\pi/2}\rangle = 0$ for $\theta = \pi/2$), the mean spin vector $\langle \boldsymbol{S}\rangle$ is in the y–z-plane. This induced the authors to consider squeezing of S_x only, as this is the *transverse* spin component coupled to radiation operators [10,17] which is also perpendicular to $\langle \boldsymbol{S}\rangle$. They considered the case $|\alpha| = 10$ and gt from 0.0 to 0.6 and reported minimum squeezing factor 0.989 (i.e. about 1.1% squeezing) at $gt = 0.22$. We repeated this, extending the ranges of gt (0 to 2) and $|\alpha|$ (0 to 27), and obtained the minimum squeezing factor $\mathsf{S}_\theta = 0.6945$ (about 30% squeezing) at $\theta = -\theta_\alpha$ or $-\theta_\alpha + \pi$, $|\alpha| = 1.45$ and $gt = 1.22$.

When we varied even $\theta + \theta_\alpha$ (i.e. when we dropped the Kitagawa-Ueda restrictions that squeezing of only spin components perpendicular to mean atomic spin will be considered) and considered all spin components, we find the minimum squeezing factor $\mathsf{S}_\theta = 0.09612$ (i.e. nearly 90% squeezing) at $\theta = -\theta_\alpha \pm \pi/2$, $gt = 0.029$ and $|\alpha| = 26.247$. In the Saito-Ueda notations ($\theta_\alpha = 0$) this corresponds to squeezing of S_y which is not perpendicular to mean atomic spin. We study variation of S_θ with gt and $|\alpha|$ near this minimum and results are given in Figures 1 and 2 respectively.

In Figure 1 we see that S_θ has two close minima at $gt = 0.02900$ and 0.03083 and a spike in between. The reason for this behaviour is that $\langle S_z\rangle$ passes through a zero at $gt \cong 0.02990$ due to Rabi oscillations and $\langle(\Delta S_\theta)^2\rangle$ also passes through a minimum value 0.001197 at $gt = 0.02988$. Consequently $\mathsf{S}_\theta \equiv \langle(\Delta S_\theta)^2\rangle/\frac{1}{2}|\langle S_z\rangle|$ (note that $\langle S_{\theta+\pi/2}\rangle = 0$) shows two minima, a lower one equal to 0.0.096121 at $gt = 0.02900$ and a relatively higher one equal to 0.099807 at $gt = 0.03083$, and a spike running to ∞ in between. The lower one is the absolute minimum. Note that a similar behaviour is seen near the next spike at $gt = 0.0897$ where $\langle S_z\rangle$ passes through second zero because of the Rabi oscillations.

It may noted that for the absolute minima of S_θ reported here mean spin vector is $\langle \boldsymbol{S}\rangle = 0.0474\hat{e}_z - 0.9972\hat{e}_\theta$, where $\hat{e}_\theta$ is unit vector in θ direction ($\hat{e}_y$ for the Saito–Ueda case) and the angle between the mean spin vector and the θ direction is nearly $\cos^{-1}(0.9983) = 3°20'29''$.

If the initial atomic state is the superradiant state $|1, 0\rangle$, our numerical results show that the minimum value

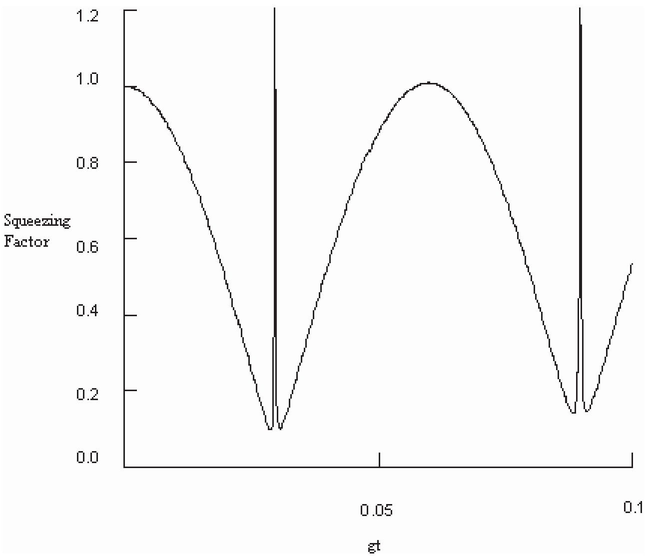

Fig. 1. Variation of squeezing factor with gt for $|\alpha| = 26.247$ and $\theta + \theta_\alpha = \pm\pi/2$ for atoms in the state $|1,1\rangle$ initially. The value of squeezing factor is >1.2 in a small regions near $gt = 0.03083$ and 0.0897.

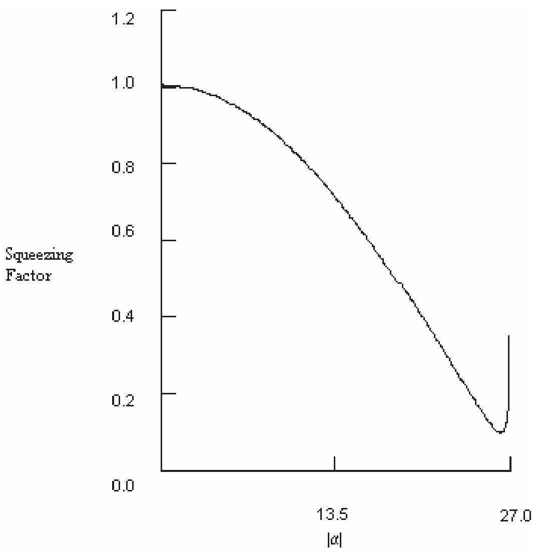

Fig. 2. Variation of squeezing factor with $|\alpha|$ for $gt = 0.029$ and $\theta + \theta_\alpha = \pm\pi/2$ for atoms in the state $|1,1\rangle$ initially.

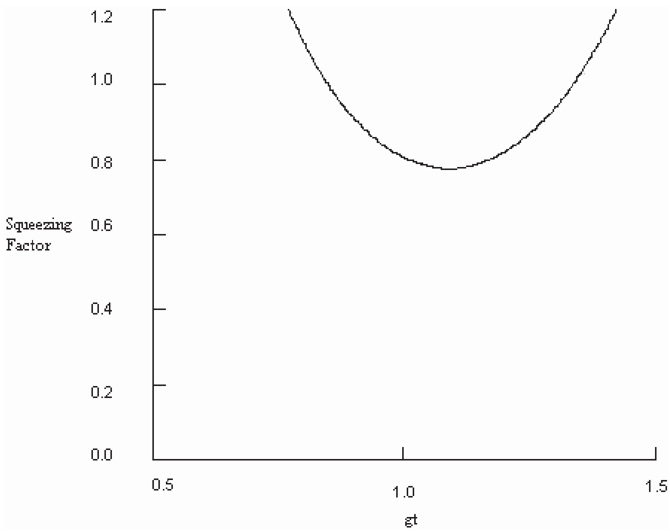

Fig. 3. Variation of squeezing factor with gt for $|\alpha| = 0.612$ and $\theta + \theta_\alpha = \pm\pi/2$ for atoms in the state $|1,0\rangle$ initially.

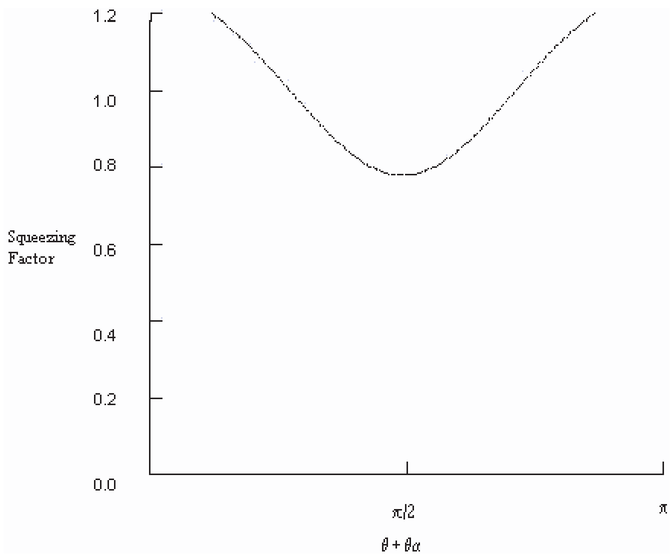

Fig. 4. Variation of squeezing factor with $\theta+\theta_\alpha$ for $|\alpha| = 0.612$ and $gt = 1.09$ for atoms in the state $|1,0\rangle$ initially.

of S_θ is equal to 0.77597 at $\theta + \theta_\alpha = \pm\pi/2$, $gt = 1.09$ and $|\alpha| = 0.612$. This gives about 22% squeezing. Variation of S_θ near this minimum value with the three independent variables gt, $\theta+\theta_\alpha$, and $|\alpha|$ are shown in Figures 3, 4 and 5 respectively. Even this squeezing is higher than that reported by Saito and Ueda but smaller than those obtained by us for the initial states $|1,\pm1\rangle$ of the atoms. For this case, mean spin vector $\langle \boldsymbol{S}\rangle = -0.7062\hat{e}_z + 0.6267\hat{e}_\theta$ and makes angle $\cos^{-1}(0.6637) = 48°24'48''$ with the θ direction

For the initial state with the two atoms in the ground state, we obtain minimum value of S_θ equal to 0.050214 at $\theta + \theta_\alpha = \pm\pi/2$, $gt = 0.071$ and $|\alpha| = 10.899$. Variation of S_θ near this minimum value with the three independent variables gt, $\theta+\theta_\alpha$, and $|\alpha|$ are shown in Figures 6, 7 and 8 respectively. It should be noted that this is the largest squeezing (nearly 95% squeezing) that we get. A further advantage of using the state $|1,-1\rangle$ over the state $|1,1\rangle$ is that no effort in preparation of the initial state of atomic assembly is required.

It should be noted that mean spin vector in this case $\langle \boldsymbol{S}\rangle = -0.02585\hat{e}_z + 0.9993\hat{e}_\theta$ and makes angle $\cos^{-1}(0.9996) = 1°28'59''$ with the θ direction. The reason behind occurrence of absolute minima at $gt = 0.071$ and a near minima at $gt = 0.0734$ and the spike at $gt = 0.07219$ is occurrence of the minimum value 0.000344 of $\langle(\Delta S_\theta)^2\rangle$ at $gt = 0.07217$ and a zero of $\langle S_z\rangle$ due to Rabi phenomenon at $gt = 0.07219$.

Usually [2,10,13] when one refers to atomic squeezing, one has in mind the relation $[S_x, S_y] = iS_z$ and

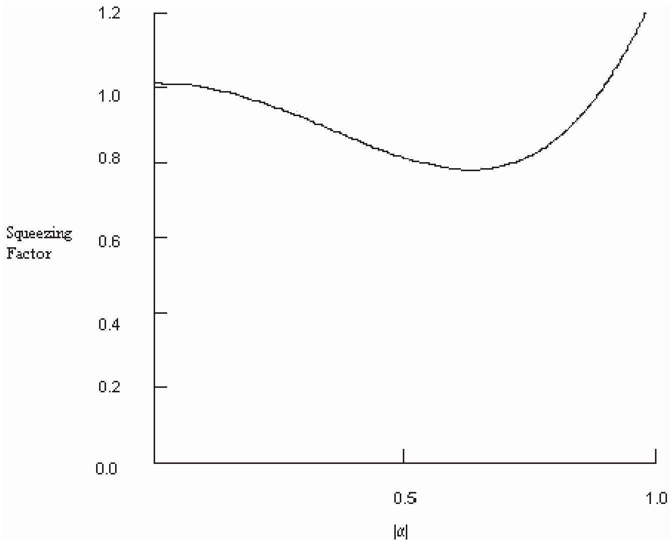

Fig. 5. Variation of squeezing factor with $|\alpha|$ for $gt = 1.09$ and $\theta + \theta_\alpha = \pm\pi/2$ for atoms in the state $|1, 0\rangle$ initially.

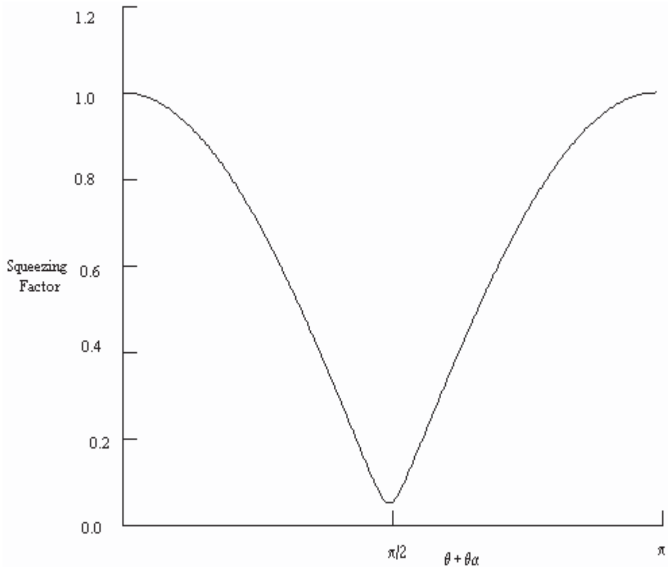

Fig. 7. Variation of squeezing factor with $\theta + \theta_\alpha$ for $|\alpha| = 10.899$ and $gt = 0.071$ for atoms in the state $|1, -1\rangle$ initially.

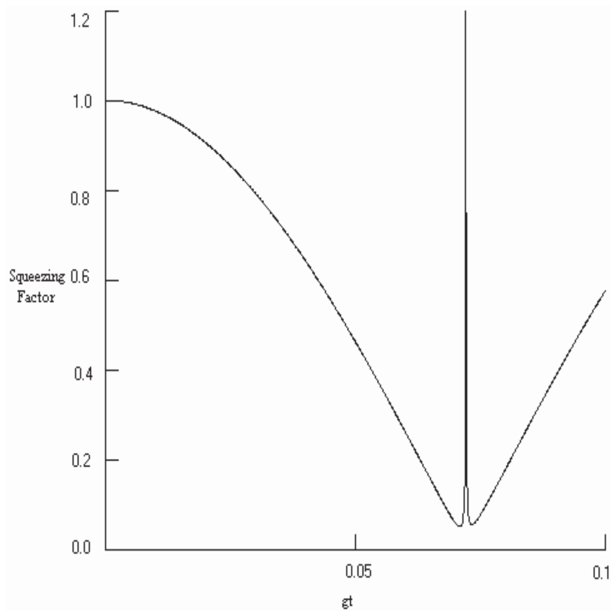

Fig. 6. Variation of squeezing factor with gt for $|\alpha| = 10.899$ and $\theta + \theta_\alpha = \pm\pi/2$ for atoms in the state $|1, -1\rangle$ initially.

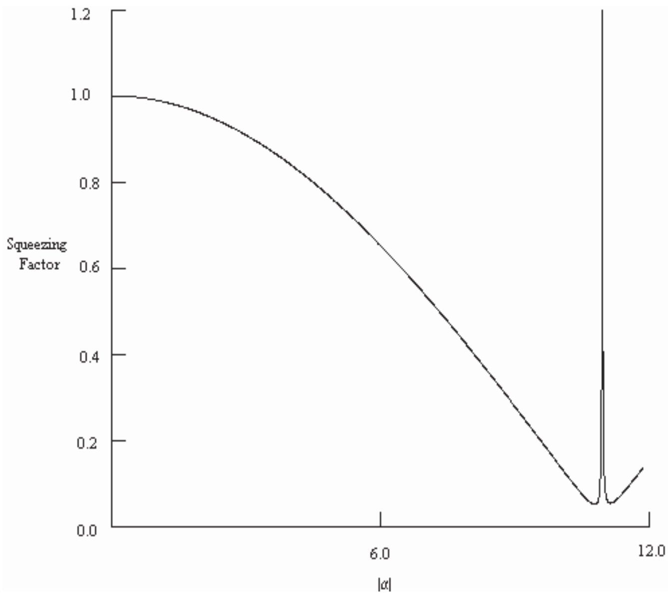

Fig. 8. Variation of squeezing factor with $|\alpha|$ for $gt = 0.071$ and $\theta + \theta_\alpha = \pm\pi/2$ for atoms in the state $|1, -1\rangle$ initially.

$\langle(\Delta S_x)^2\rangle\langle(\Delta S_y)^2\rangle \geqslant \frac{1}{4}|\langle S_z\rangle|^2$, and therefore only one of the two orthogonal components S_x and S_y has variance less than $\frac{1}{2}|\langle S_z\rangle|$ and is regarded squeezed. However, if one uses, the more general definition equation (14) for squeezing orthogonal components S_θ and $S_{\theta+\pi/2}$ may be regarded squeezed simultaneously in the general sense (i.e. with different sets of triads of operators) and not in the usual sense (i.e. with the same triad), if [20]

$$\left\langle(\Delta S_\theta)^2\right\rangle < \frac{1}{2}\sqrt{\left(\langle S_z\rangle^2 + \left\langle S_{\theta+\pi/2}\right\rangle^2\right)};$$

$$\left\langle(\Delta S_{\theta+\pi/2})^2\right\rangle < \frac{1}{2}\sqrt{\left(\langle S_z\rangle^2 + \langle S_\theta\rangle^2\right)}.$$

In Figure 7, we note another case where, all possible pairs of orthogonal components, S_θ and $S_{\theta+\pi/2}$ are squeezed, except those for which $\theta + \theta_\alpha = 0, \pm\pi/2, \pm\pi$ and for these one component is squeezed but the other has $\mathsf{S}_\theta = 1$. Another example of this kind was reported by us earlier [20].

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References

1. See, e.g., the review articles, D.F. Walls, Nature **306**, 141 (1983); R. Loudon, P.L. Knight, J. Mod. Opt. **34**, 709 (1987); V.V. Dodonov, J. Opt. B **4**, R1 (2002)
2. See, e.g., M.O. Scully, M.S. Zubairy, *Quantum Optics* (Cambridge University Press, 1997), pp. 56-65; Jin Sheng Peng, Gao-Xiang Li, *Introduction to Modern Quantum Optics* (World Scientific, 1999), pp. 361-379
3. B.R. Mollow, R.J. Glauber, Phys. Rev. **160**, 1076 (1967)
4. H. Prakash, N. Chandra, Lett. Nuovo Cim. **4**, 1196 (1970); H. Prakash, N. Chandra, Ind. J. Pure Appl. Phys. **9**, 677 (1971); H. Prakash, N. Chandra, Lett. Nuovo Cim. **9**, 688 (1971); H. Prakash, N. Chandra, Lett. Nuovo Cim. **9**, 767 (1971)
5. See, e.g., H.-A. Bachor, *A Guide to Experiments in Quantum Optics* (Wiley-VCH, 1998), pp. 223-230
6. H. Takahashi, in *Advances in Communication Systems*, edited by A.V. Balakrishnan (Academic Press, Orlando FL, 1965), pp. 227; H.P. Yuen, J.H. Shapiro, IEEE Trans. Inform. Theory **IT-25**, 657 (1978); H.P. Yuen, J.H. Shapiro, IEEE Trans. Inform. Theory **IT-26**, 78 (1980); Y. Yamamoto, H.A. Haus, Rev. Mod. Phys. **58**, 1001 (1986); R.E. Slusher, B. Yurke, J. Lightwave Technol. **8**, 466 (1990)
7. C.M. Caves, Phys. Rev. D **23**, 1639 (1981); R. Loudan, Phys. Rev. Lett. **47**, 815 (1981); "*Quantum Optics, Experimental Gravitation, and Measurement Theory*", edited by P. Meystre, M.O. Scully (Plenum Press, New York, 1981), NATO ASI Series B: Physics **94**; see articles by K.S. Thorne (pp. 325-346), R.W.P. Drever and colleagues (pp. 503-514), J. Hough, and colleagues (pp. 515-524)
8. L. Vaidman, Phys. Rev. A **49**, 1473 (1994); A. Furusawa et al., Science **282**, 706 (1998); S.L. Braunstein, H.J. Kimble, Phys. Rev. Lett. **80**, 869 (1998); S.L. Braunstein, A.K. Pati, *Quantum Information with Continuous Variables* (Klauwer Academic, Dordrecht, 2003); K. Hammerer, E.S. Polzik, J.I. Cirac, Phys. Rev. A **72**, 052313 (2005)
9. H. Prakash, N. Chandra, Vachaspati, Phys. Rev. A **9**, 2167 (1974); H. Prakash, N. Chandra, Vachaspati, Ind. J. Pure Appl. Phys. **13**, 757 (1976); H. Prakash, N. Chandra, Vachaspati, Ind. J. Pure Appl. Phys. **13**, 763 (1976); H. Prakash, N. Chandra, Vachaspati, Ind. J. Pure Appl. Phys. **14**, 41 (1976); H. Prakash, N. Chandra, Vachaspati, Ind. J. Pure Appl. Phys. **14**, 48 (1976); M.T. Raiford, Phys. Rev. A **9**, 2060 (1974); G. Milburn, D.F. Walls, Opt. Commun. **39**, 401 (1981); Ling-Au, Wu, H.J. Kimble, J.L. Hall, Huifa Wu, Phys. Rev. Lett. **57**, 2520 (1986)
10. D.F. Walls, P. Zoller, Phys. Rev. Lett. **47**, 709 (1981)
11. P.A. Meystre, M.S. Zubairy, Phys. Lett. **89A**, 390 (1982)
12. G.S. Agrawal, R.R. Puri, Phys. Rev. A **41**, 3782 (1990)
13. K. Wodkiewicz, Phys. Rev. B **32**, 4750 (1985); K. Wodkiewicz, J.H. Eberly, J. Opt. Soc. Am. B **2**, 458 (1985); S.M. Barnett, M.A. Dupertuis, J. Opt. Soc. Am. B **4**, 505 (1987)
14. J. Seke, J. Opt. B: Quantum Semiclass. Opt. **7**, 161 (1995)
15. D.J. Wineland, J.J. Bollinger, W.M. Itano, F.L. Moore, Phys. Rev. A **46**, R6797 (1992)
16. M. Kitagawa, M. Ueda, Phys. Rev. A **47**, 5138 (1993)
17. H. Saito, M. Ueda, Phys. Rev. A **59**, 3959 (1999)
18. A. Sorensen, L.M. Duan , J.I. Cirac, P. Zoller, Nature **409**, 63 (2001)
19. X. Wang, B.C. Sanders, Phys. Rev. A **68**, 033821 (2003)
20. H. Prakash, R. Kumar, J. Opt B: Quantum Semiclass. Opt. **7**, S757 (2005)
21. J. Hald, J.L. Sorensen, C. Schori, E.S. Polzik, Phys. Rev. Lett. **83**, 1319 (1999); A. Kuzmich, K. Molmer, E.S. Polzik, Phys. Rev. Lett. **79**, 4782 (1997)
22. See e.g., L. Allen, J.H. Eberly, *Optical Resonance and Two-Level Atoms* (John Wiley and Sons Inc., 1975), pp. 34-45; E.T. Jaynes, F.W. Cummings, Proc. IEEE **51**, 89 (1963); M. Tavis, F.W. Cummings, Phys. Rev. **170**, 379 (1968); H.I. Yoo, J.H. Eberly, Phys. Rep. **118**, 239 (1985); V. Buzek, J. Mod. Opt. **37**, 1033 (1990); B.W. Shore, P.L. Knight, J. Mod. Opt. **40**, 1195 (1993)
23. R.H. Dicke, Phys. Rev. **93**, 99 (1954)
24. S. Stenholm, Phys. Rep. **6**, 1 (1973)
25. H. Prakash, N. Chandra and Vachaspati, Acta Phys. Pol. A **48**, 123 (1975)
26. Note that $|\langle \boldsymbol{S} \rangle| = \sqrt{\langle S_x \rangle^2 + \langle S_y \rangle^2 + \langle S_z \rangle^2}$; $\langle \boldsymbol{S} \rangle^2 = \langle \boldsymbol{S} \cdot \hat{\mathbf{n}} \rangle^2 + |\langle \boldsymbol{S} \times \hat{\mathbf{n}} \rangle|^2 \geqslant \langle \boldsymbol{S} . \hat{\mathbf{n}} \rangle^2$
27. Reference [17], equation (21) on page 3962